| Question |  | Answer |  | Guidance |  |
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| 1 |  | $\begin{aligned} \int_{1}^{2} \frac{1}{\sqrt{3 x-2}} \mathrm{~d} x & =\left[\frac{2}{3}(3 x-2)^{1 / 2}\right]_{1}^{2} \\ & =\frac{2}{3} \cdot 2-\frac{2}{3} \cdot 1 \\ & =2 / 3^{*} \end{aligned}$ <br> OR $\begin{gathered} u=3 x-2 \Rightarrow \mathrm{~d} u / \mathrm{d} x=3 \\ \Rightarrow \quad \int_{1}^{2} \frac{1}{\sqrt{3 x-2}} \mathrm{~d} x=\int_{1}^{4} \sqrt{u} \cdot \frac{1}{3} \mathrm{~d} u \\ =\left[\frac{2}{3} u^{1 / 2}\right]_{1}^{4}=\frac{2}{3} \cdot 2-\frac{2}{3} \cdot 1 \\ =2 / 3^{*} \end{gathered}$ | M1 A2 M1dep A1 M1 A1 A1 M1dep A1 [5] | $\begin{aligned} & {\left[\begin{array}{l} {\left[(3 x-2)^{1 / 2}\right]} \\ k=2 / 3 \\ \text { substituting limits dep } 1^{\text {st }} \mathrm{M} 1 \\ \text { NB AG } \\ \int \frac{1}{\sqrt{u}} \\ \times 1 / 3(\mathrm{~d} u) \\ {\left[\frac{2}{3} u^{1 / 2}\right] \mathrm{o} .} \end{array} .\right.} \end{aligned}$ <br> substituting correct limits dep $1^{\text {st }}$ M1 NB AG | $\begin{aligned} & \text { or } w^{2}=3 x-2 \Rightarrow \int \frac{1}{w} \\ & \times 2 / 3 w(\mathrm{~d} w) \\ & {\left[\frac{2}{3} w\right]} \end{aligned}$ <br> upper - lower, 1 to 4 for $u$ or 1 to 2 for $w$ or substituting back (correctly) for $x$ and using 1 to 2 |


| Question |  | Answer | Marks | Guidance |  |
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| 2 | (i) | (A) $(0,6)$ and $(1,4$ <br> (B) $-1,5)$ and $(0,4)$ | $\begin{gathered} \text { B1B1 } \\ \text { B1B1 } \\ \text { [4] } \end{gathered}$ | Condone P and Q incorrectly labelled (or unlabelled) |  |
|  | (ii) | $\begin{aligned} & \mathrm{f}^{\prime}(x)=\frac{(x+1) \cdot 2 x-\left(x^{2}+3\right) \cdot 1}{(x+1)^{2}} \\ & \mathrm{f}^{\prime}(x)=0 \Rightarrow 2 x(x+1)-\left(x^{2}+3\right)=0 \\ & \Rightarrow x^{2}+2 x-3=0 \\ & \Rightarrow(x-1)(x+3)=0 \\ & \Rightarrow x=1 \text { or } x=-3 \end{aligned}$ <br> When $x=-3, y=12 /(-2)=-6$ <br> so other TP is $(-3,-6)$ | A1 <br> M1 A1dep <br> B1B1cao | Quotient or product rule consistent with their derivatives, condone missing brackets <br> correct expression <br> their derivative $=0$ <br> obtaining correct quadratic equation (soi) dep $1^{\text {st }}$ M1 but withhold if denominator also set to zero <br> must be from correct work (but see note re quadratic) | PR: $\left(x^{2}+3\right)(-1)(x+1)^{-2}+2 x(x+1)^{-1}$ If formula stated correctly, allow one substitution error. condone missing brackets if subsequent working implies they are intended Some candidates get $x^{2}+2 x+3$, then realise this should be $x^{2}+2 x-3$, and correct back, but not for every occurrence. Treat this sympathetically. <br> Must be supported, but - 3 could be verified by substitution into correct derivative |
|  | (iii) | $\begin{aligned} \mathrm{f}(x-1) & =\frac{(x-1)^{2}+3}{x-1+1} \\ & =\frac{x^{2}-2 x+1+3}{x-1+1} \\ & =\frac{x^{2}-2 x+4}{x}=x-2+\frac{4}{x} * \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \\ & \text { A1 } \\ & \text { [3] } \end{aligned}$ | substituting $x-1$ for both $x$ 's in f <br> NB AG | allow 1 slip for M1 |
|  | (iv) | $\begin{aligned} & \int_{a}^{b}\left(x-2+\frac{4}{x}\right) \mathrm{d} x=\left[\frac{1}{2} x^{2}-2 x+4 \ln x\right]_{a}^{b} \\ & =\left(\frac{1}{2} b^{2}-2 b+4 \ln b\right)-\left(\frac{1}{2} a^{2}-2 a+4 \ln a\right) \end{aligned}$ <br> Area is $\int_{0}^{1} \mathrm{f}(x) \mathrm{d} x$ <br> So taking $a=1$ and $b=2$ $\begin{gathered} \text { area }=(2-4+4 \ln 2)-(1 / 2-2+4 \ln 1) \\ =4 \ln 2-1 / 2 \end{gathered}$ | $\begin{gathered} \text { B1 } \\ \text { M1 } \\ \text { A1 } \\ \\ \text { M1 } \\ \\ \text { A1 cao } \\ \text { [5] } \\ \hline \end{gathered}$ | $\left[\frac{1}{2} x^{2}-2 x+4 \ln x\right]$ <br> $\mathrm{F}(b)-\mathrm{F}(a)$ condone missing brackets oe (mark final answer) <br> must be simplified with $\ln 1=0$ | F must show evidence of integration of at least one term <br> or $\mathrm{f}(x)=x+1-2+4 /(x+1)$ $\begin{aligned} & A=\int_{0}^{1} \mathrm{f}(x) \mathrm{d} x=\left[\frac{1}{2} x^{2}-x+4 \ln (1+x)\right]_{0}^{1} \mathrm{M} 1 \\ & =1 / 2-1+4 \ln 2=4 \ln 2-1 / 2 \mathrm{~A} 1 \end{aligned}$ |


| 3 | $\int_{0}^{\pi / 6} \sin 3 x \mathrm{~d} x=\left[-\frac{1}{3} \cos 3 x\right]_{0}^{\frac{\pi}{6}}$ | B1 |
| :--- | :--- | :--- |
| $=-\frac{1}{3} \cos \frac{\pi}{2}+\frac{1}{3} \cos 0$ | M1 | $\left[-\frac{1}{3} \cos 3 x\right]$ or $\left[-\frac{1}{3} \cos u\right]$ |
| $=\frac{1}{3}$ | A1cao <br> $[3]$ | 0.33 or better. |


| 4(i) $y=1 /(1+\cos \pi / 3)=2 / 3$. | $\begin{aligned} & \text { B1 } \\ & {[1]} \end{aligned}$ | or 0.67 or better |
| :---: | :---: | :---: |
| (ii) $\begin{aligned} f^{\prime}(x)= & -1(1+\cos x)^{-2} .-\sin x \\ & =\frac{\sin x}{(1+\cos x)^{2}} \end{aligned}$ <br> When $x=\pi / 3, \mathrm{f}^{\prime}(x)=\frac{\sin (\pi / 3)}{(1+\cos (\pi / 3))^{2}}$ $=\frac{\sqrt{3} / 2}{\left(1 \frac{1}{2}\right)^{2}}=\frac{\sqrt{3}}{2} \times \frac{4}{9}=\frac{2 \sqrt{3}}{9}$ | M1 <br> B1 <br> A1 <br> M1 <br> A1 <br> [5] | chain rule or quotient rule $\mathrm{d} / \mathrm{d} x(\cos x)=-\sin x$ soi correct expression substituting $x=\pi / 3$ oe or 0.38 or better. ( $0.385,0.3849$ ) |
| $\begin{aligned} \text { (iii) deriv } & =\frac{(1+\cos x) \cos x-\sin x \cdot(-\sin x)}{(1+\cos x)^{2}} \\ & =\frac{\cos x+\cos ^{2} x+\sin ^{2} x}{(1+\cos x)^{2}} \\ & =\frac{\cos x+1}{(1+\cos x)^{2}} \\ & =\frac{1}{1+\cos x} * \\ \text { Area } & =\int_{0}^{\pi / 3} \frac{1}{1+\cos x} d x \\ & =\left[\frac{\sin x}{1+\cos x}\right]_{0}^{\pi / 3} \\ & =\frac{\sin \pi / 3}{1+\cos \pi / 3}(-0) \\ & =\frac{\sqrt{3}}{2} \times \frac{2}{3}=\frac{\sqrt{3}}{3} \end{aligned}$ |  | Quotient or product rule condone uv' - u’v for M1 correct expression $\cos ^{2} x+\sin ^{2} x=1$ used dep M1 www substituting limits or $1 / \sqrt{ } 3$ - must be exact |
| $\begin{array}{ll} \text { (iv) } & y=1 /(1+\cos x) \quad x \leftrightarrow y \\ & x=1 /(1+\cos y) \\ \Rightarrow & 1+\cos y=1 / x \\ \Rightarrow & \cos y=1 / x-1 \\ \Rightarrow & y=\arccos (1 / x-1)^{*} \end{array}$ <br> Domain is $1 / 2 \leq x \leq 1$ | M1 <br> A1 <br> E1 <br> B1 <br> B1 <br> [5] | attempt to invert equation <br> www <br> reasonable reflection in $y=x$ |

